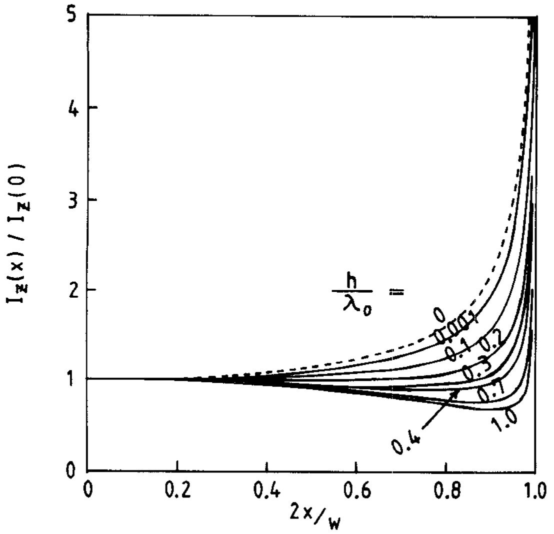

Fig. 5. Frequency-dependent characteristics of normalized longitudinal current distributions ($\epsilon = 8$, $w/h = 1$). --- Green's function technique [11]; — present method.

The shifts of the current distributions with respect to frequencies for $h/\lambda_0 \leqslant 0.2$ shown in Figs. 4 and 5 are similar to those revealed by Shih *et al.* [13], although for cases where ϵ and w/h have values different from those of the present article.

IV. Conclusion

The spectral-domain approach has been used to obtain the frequency-dependent characteristics of current distributions and the effective permittivities of open microstrip lines. The functions $U_{2n}(2x/w)$ and $T_{2(n-1)}(2x/w)/\sqrt{1-(2x/w)^2}$ have been adopted as basis functions; $T_n(x)$ and $U_n(x)$ are Chebyshev polynomials of the first and second kinds, respectively. Numerical results reported in this article have been compared with other available data.

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Shift of the Complex Resonance Frequency of a Dielectric-Loaded Cavity Produced by Small Sample Insertion Holes

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Abstract —**The presence of small sample insertion holes in a cylindrical cavity produces a shift in the complex resonance frequency of the cavity. A mathematical model is proposed to compute the shift when the cavity oscillates in an axially symmetric TM_{0mp} mode. The treatment applies to samples with arbitrary complex permittivity. The model is compared with other treatments and checked against measured results.**

I. Introduction

Insertion holes in resonant cavities produce changes in both the real and imaginary parts of the complex resonance frequency, which may amount to a few percent and are significant in high-precision measurements. Several attempts have been made to quantify hole effects. Estin and Bussey [1] and Meyer [2] have estimated the change in the real part of the resonance frequency for some simple TM_{0mp} modes. Their main assumptions were that the field is not perturbed in the main body of the cavity and that in the tubes it is well represented by the first evanescent TM mode. More recently, Li and Bosisio [3] have significantly improved the treatment by allowing for a large number of modes in the tubes. They have obtained correction terms due to insertion holes for both the real part of the resonance frequency and the quality factor of the cavity.

The present paper is an attempt to compute the shift of the complex resonance frequency of a cavity produced by small sample insertion holes. It was largely inspired by the work of Li and Bosisio, which it tries to improve in two different ways. First, we take fully into account the fact that, for lossy samples, the phasors and the wavenumbers in the tubes are genuinely complex. Second, we carry a larger fraction of the calculations analytically. The resulting formulas are less susceptible to numerical errors.

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The field distribution in the tubes is obtained in Section II. In Section III, we derive correction terms to the complex resonance frequency of the cavity due to the presence of the holes. Finally, results are checked against experimental measurements and discussed in Section IV.

II. Field Distribution

Fig. 1 illustrates a cylindrical cavity enclosed in perfectly conducting walls. The cavity is symmetrical about the z axis and the plane $z=-h/2$. Its main body has radius R_{N+1} and height h. Two smaller tubelike end pieces have radii R_N. The cavity is filled with concentric dielectric samples of radii $R_1, R_2, \cdots, R_{N+1}$ and complex permittivity $\epsilon_1, \epsilon_2, \cdots, \epsilon_{N+1}$. Usually $\epsilon_{N+1}=\epsilon_0$, the permittivity of free space. We assume that the samples' permeability is equal to μ_0 throughout.

In experimental setups, where the cavity is used to make permittivity measurements, N rarely exceeds 3. Furthermore, $R_N \ll R_{N+1}$. This implies that the fields in the tubes decrease exponentially for all frequencies of interest. The height of the tubes is much larger than R_N and can therefore be taken equal to infinity.

With no end pieces, the cavity sustains TE and TM modes whose mathematical expression is known exactly [4]. We assume the cavity has been so excited that the electromagnetic field in its main body is well represented by a single TM_{0mp} mode. By continuity, the fields in the end pieces will also be transverse magnetic and have cylindrical symmetry. In the upper end piece, the most general exponentially decreasing form for these fields is given by

$$E_z = \sum_{\nu=1}^{\infty} A_\nu Z_0(k_\nu r)\exp(-\gamma_\nu z) \tag{1}$$

$$E_r = \sum_{\nu=1}^{\infty} A_\nu \frac{\gamma_\nu}{k_\nu} Z_1(k_\nu r)\exp(-\gamma_\nu z) \tag{2}$$

$$H_\phi = \sum_{\nu=1}^{\infty} A_\nu \frac{j\omega\epsilon}{k_\nu} Z_1(k_\nu r)\exp(-\gamma_\nu z) \tag{3}$$

$$E_\phi = H_z = H_r = 0. \tag{4}$$

Each A_ν is a constant. The parameters k_ν and γ_ν satisfy the relation

$$k_\nu^2 = \gamma_\nu^2 + \omega^2\mu_0\epsilon \tag{5}$$

where ω is the angular frequency of the cavity. The parameter γ_ν does not vary from sample to sample but, in general, the parameter k_ν does through its dependence on ϵ. The function Z_0 (Z_1) is a linear combination of Bessel functions J_0 and Y_0 (J_1 and Y_1) of the same argument. We have $-Z_1 = Z_0'$. For $r<R_1$, Z_0 can be set equal to J_0. Where two samples meet, Z_0 and $(\epsilon/k_\nu)Z_1$ are continuous. The circular walls make $Z_0(k_\nu R_N)$ vanish.

We shall now make an important approximation. We assume that the radius of the tubes (R_N) is much smaller than the linear dimensions of the main body of the cavity (R_{N+1} and h). We also assume that the indices p and m of the TM_{0mp} mode are not too large. Since $k_\nu R_N$ is a zero of Z_0, $|k_\nu R_N| \gtrsim 1$, so that k_ν^2 is at least of order $(R_N)^{-2}$. On the other hand, $\omega^2\mu_0\epsilon_{N+1}$ is of order $(R_{N+1})^{-2}$ (or h^{-2}). From (5), we thus make the approximation $\gamma_\nu = k_\nu$, for every ν, in (1)–(3).

The coefficients A_ν in (1)–(3) are still not determined. Suppose that a component of the field (say E_z) is known on the surface

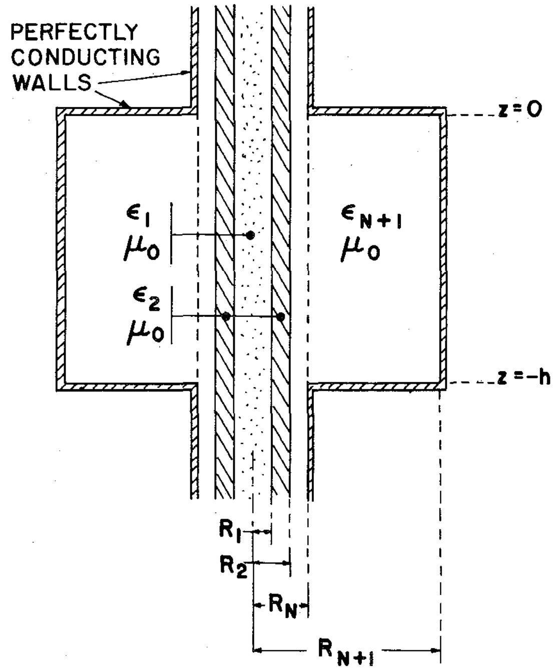

Fig. 1. The cylindrical cavity with concentric dielectric samples and thin, infinitely long insertion tubes.

$z=0$, that is, on the interface between the tube and the cavity. The series in (1) is a Fourier–Bessel series and hence the coefficients A_ν can be expressed as [5], [6]

$$A_\nu = \frac{\int_0^{R_N} r\epsilon Z_0(k_\nu r)\,E_z(z=0)\,dr}{\int_0^{R_N} r\epsilon Z_0(k_\nu r)\,Z_0(k_\nu r)\,dr}. \tag{6}$$

Knowledge of $E_z(z=0)$ thus completely determines all the A_ν's.

As an illustration, we will assume that E_z is not perturbed, on the plane $z=0$, by the presence of the protruding end piece. (This is equivalent to the assumption, made in [3], that H_ϕ is not perturbed.) In the plane $z=0$, E_z is given for $r<R_N$ by $E_z(z=0)=A$, where A is a constant and terms of order $\epsilon R_N^2/\epsilon_{N+1}R_{N+1}^2$ have been neglected. The coefficients A_ν can now be determined from (6). We easily get

$$A_\nu = \frac{A\epsilon_N R_N Z_1(k_\nu R_N)}{k_\nu \int_0^{R_N} r\epsilon[Z_0(k_\nu r)]^2\,dr}. \tag{7}$$

The integral in the denominator can be computed exactly from well-known properties of Bessel functions [5], [6]. The result is

$$\int_0^{R_N} r\epsilon[Z_0(k_\nu r)]^2\,dr = \frac{\epsilon_N R_N^2}{2}[Z_1(k_\nu R_N)]^2 - \sum_{i=1}^{N-1}\frac{R_i^2}{2}\left[\left(\frac{1}{\epsilon_{i+1}}-\frac{1}{\epsilon_i}\right)[\epsilon_i Z_1(k_\nu R_i)]^2 + (\epsilon_{i+1}-\epsilon_i)[Z_0(k_\nu R_i)]^2\right]. \tag{8}$$

To interpret (8), recall from Fig. 1 that ϵ_i is the permittivity in the region $R_{i-1}<r<R_i$. In the product $\epsilon_i Z_1(k_\nu R_i)$, the limit $r\to R_i$ should be taken from the region of permittivity ϵ_i.

III. Complex Resonance Frequency

The complex resonance frequency ω of any cavity can be written as

$$\omega = \omega_r(1 + j/2Q) \tag{9}$$

where ω_r is the real part of ω and Q is the quality factor. For the ideal cavity, the resonance frequency ω of each TM_{0mp} mode is known exactly. For the cavity with thin, infinitely long tubes, depicted in Fig. 1, ω is shifted by a small quantity $\delta\omega$. Assuming that the change in the fields is negligible in the main body of the cavity, we can use perturbation methods [4] and compute $\delta\omega$ as

$$\frac{\delta\omega}{\omega} = \frac{\delta\omega_r}{\omega_r} + \frac{j}{2}\delta\left(\frac{1}{Q}\right) = \frac{1}{W_c}\iiint_{V_{\text{tube}}} \epsilon^* \boldsymbol{E}\cdot\boldsymbol{E}^*\, dV. \tag{10}$$

Here the integral is on one tube only and W_c is the total energy in the cavity.

We now substitute (1) and (2) (with $\gamma_\nu = k_\nu$) in the numerator of (10). The integral can be evaluated exactly through the use of well-known properties of Bessel functions [5], [6]. The result can be written as

$$\frac{\delta\omega}{\omega} = \frac{1}{W_c}\sum_{\nu=1}^{\infty}\sum_{\mu=1}^{\infty} A_\nu A_\mu^* M_{\mu\nu} \tag{11}$$

where

$$M_{\mu\nu} = \frac{2\pi}{(k_\mu^*)^2 - (k_\nu)^2}\sum_{i=1}^{N-1} R_i\left(\frac{\epsilon_{i+1}^*}{\epsilon_{i+1}} - \frac{\epsilon_i^*}{\epsilon_i}\right) \cdot\left[Z_0(k_\mu R_i)\right]^*\left[\epsilon_i Z_1(k_\nu R_i)\right]. \tag{12}$$

This expression is indeterminate if the permittivity is everywhere real. In that case we find that $M_{\mu\nu} = 0$ if $\nu \neq \mu$ and

$$M_{\nu\nu} = \frac{2\pi}{k_\nu}\int_0^{R_N} r\epsilon\left[Z_0(k_\nu r)\right]^2 dr. \tag{13}$$

This integral is calculated in (8).

IV. Results and Discussion

When a resonant cavity is used to make complex permittivity measurements, the complex frequency differences are the parameters whose accurate evaluation is particularly important. A cavity with insertion holes, such as the one depicted in Fig. 1, produces a spurious frequency shift both when empty and when containing dielectric material. It is the difference in these shifts that is especially relevant to permittivity measurements.

To illustrate hole effects, a configuration with three distinct regions will be considered. A sample of complex permittivity ϵ_1 and radius R_1 is contained in a capillary tube of real permittivity $\epsilon_2 = 4.75$ (a typical value for Pyrex glass) and radius R_2. The radius of the insertion holes is R_3, and the permittivity outside the capillary is equal to ϵ_0. Let $(\delta\omega)_{\text{sample}}$ be the complex frequency shift due to the holes, which is calculated by (11), for the configuration just described. Let $(\delta\omega)_{\text{tube}}$ be the shift for the configuration where only the capillary tube is present (that is, the sample is removed). Finally, let $\Delta\omega$ be the difference between the resonant frequencies of an ideal cavity with and without sample.

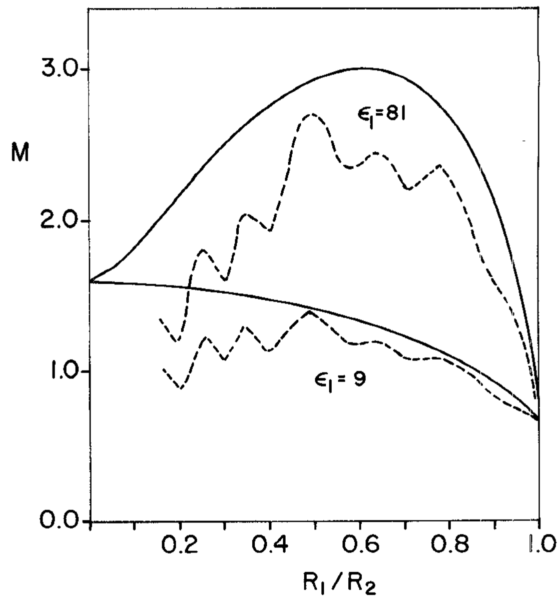

Fig. 2. The coefficient M versus R_1/R_2 for $\epsilon_1 = 9$ and $\epsilon_1 = 81$ ($\epsilon_2 = 4.75$, $R_2 = R_3 = 1$ mm) Broken lines ref. [3]; solid lines: present analysis

TABLE I
Values of ϵ'' for Several Materials, Corrected as in [3], [7] and Herein ($R_3 = 1$ mm, $f = 2.23$ GHz, $T = 22°$C)

Material	R_1 (mm)	R_2 (mm)	ϵ''_{unc}	ϵ''_{cor} [3] [7]	ϵ''_{cor} (here)	Reference value	Reference
Methanol	.304	.749	13.4	13.1	13.0	12.5-13.5	[8]
1-Propanol	.305	.756	3.66	3.63	3.58	1.45-2.81	[9]
1-Buthanol	.691	.959	2.02	1.96	1.99	0.87-1.64	[9]
Water	.304	.700	8.80	8.62	8.41	8-12.5	[9, 10, 4]
	.432	.708	8.19	8.03	7.73		

We define

$$M = \frac{h}{R_3}\,\frac{\text{Re}(\delta\omega)_{\text{tube}} - \text{Re}(\delta\omega)_{\text{sample}}}{\text{Re}(\Delta\omega)}. \tag{14}$$

M represents the relative error on the real part of the sample's permittivity ϵ_1 produced by the insertion holes. It is plotted in Fig. 2 for two values of ϵ_1, together with similar curves taken from [3]. We believe the solid lines are smoother because the more complete analytical treatment has eliminated numerical instabilities.

Corrections to the imaginary part of the permittivity are illustrated in Table I. Experimental results presented in [7] are analyzed according to [3] and to the methods presented here, and compared with reference values.

Although our computation of the field distribution in the insertion tubes rests on the assumption that E_z is not changed at the interface $z = 0$, it can be adapted to other choices of E_z ($z = 0$). The coefficients A_ν are then given by (6). In fact, the value of the fields in the vicinity of the insertion tubes, and in particular of E_z at the interface, can be determined by a computer simulation of the field distribution in and near the tube. Work in that direction is in progress.

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